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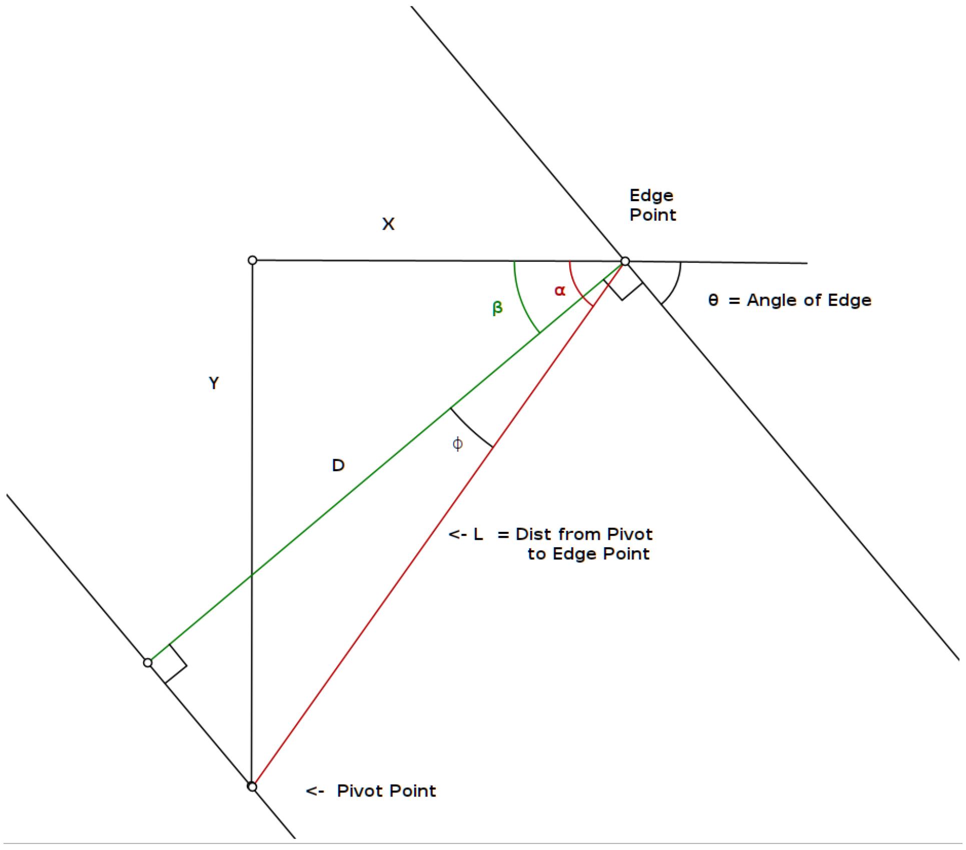
In terms the variables O for offset between the mounting plane and the pivot point (ie. along the square rail's direction in the sharpener), X and Y for the pure horizontal and pure vertical positions of the blade edge point being considered relative to that pivot (the X and Y directions define what we will call the mounting plane, this plane 'lives' in the center of the knife clamp), and finally θ for the angle that the blade edge makes at that point (when mounted), we can determine the sharpening angle there as:

$$tan^{-1}\left[\frac{O}{\sqrt{X^2+Y^2}*\cos\left(\left|sin^{-1}\left(\frac{Y}{\sqrt{(X^2+Y^2)}}\right)-\{90-sign(X)*\theta\}\right|\right)}\right]$$

And yes, this is pretty opaque. It is MUCH easier to see how the analysis goes if we do this step by step using angles from the following diagram which represents info from the mounting plane:



In this diagram L is the distance from the edge point (where we are evaluating) to the pivot point, again, considering ONLY the in-plane horizontal and vertical directions. These are in the figure as the line lengths X and Y respectively. (Remember that the real pivot point is a distance O outside the mounting plane). To begin

$$L=\sqrt{(X^2+Y^2)}$$

In the figure, θ shows the edge angle as below the horizontal at the edge point. If it is angling DOWN, we consider it in the positive direction. (An upward blade angle is then negative). There is a line passing through the edge point in the θ direction, and another line parallel to that which passes thru the projection of the pivot point in this XY plane. The distance between these two lines along the direction perpendicular both of them, D, is the key distance for our computation. The sharpening angle at any knife position (X, Y) is:

$$\text{Sharpening Angle} = tan^{-1}\left(\frac{O}{D}\right)$$

(Where O is not shown in the figure).

There are a number of ways to justify this. Remember that a flat blade at angle θ = zero has a uniform sharpening angle. That geometry would give a value of D that is the same everywhere along the blade. The same is true, as can be deduced from the figure, that D is the same for every XY point along a flat blade at an arbitrary but uniform θ, another geometry which is known to have a uniform sharpening angle. A third uniform sharpening angle geometry is an edge that lies along a circle centered at the pivot point. In that case, as well, D is uniform (in fact D = L). (To shown that this distance D is what we need for an arbitrary knife geometry can be inferred from the discussion of Anthony Yan, especially by a reconsideration of his Figure 6.8. The difference we consider is the plane formed by the line at the knife edge angle in the XY plane, and by the line parallel to it that passes through the actual pivot point. The sharpening angle is made by the line that lives in this new plane that is **perpendicular** to the knife edge angle line.) (Wish I could easily make a 3-D drawing here...)

We need, therefore, to calculate D. From the figure of the mounting plane it is can be deduced that $D = L * cos(\phi)$, where ϕ is the angle between D and L. The angle ϕ can be determined from the angles θ (the blade angle), β, and α. Again from the figure, β is just $90-\theta$ (since those two sum to a right angle). The angle α is the angle that L makes below the horizontal, which is

$$\alpha = sin^{-1}\left(\frac{X}{L}\right)$$

. And finally $\phi = \alpha - \beta$.

There are in fact six different possible geometric arrangements, the geometry shown in the figure is the simplest. All of these reduce to the same result, however, if we use $\beta = 90 - sign(X) * \theta$. Combining these angle based equations in terms of the variables X, Y, θ and the offset O leads to the overall equation shown at the start.